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Theoretical Determination of the $\Delta N\gamma$ Electromagnetic Transition Amplitudes in the $\Delta(1232)$ Region

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Abstract

We utilize non-perturbative and fully relativistic methods to calculate the $\Delta N\gamma$ electromagnetic transition amplitudes $G_M^*(q^2)$ (related to the magnetic dipole moment $M_{1^+}^{3/2}(q^2)$), $G_E^*(q^2)$ (related to the electric quadrupole moment $E_{1^+}^{3/2}(q^2)$), the electromagnetic ratio $R_{EM}(q^2) \equiv -G_E^*(q^2)/G_M^*(q^2) = E_{1^+}^{3/2}(q^2)/M_{1^+}^{3/2}(q^2)$, and discuss their q^2 behavior in the $\Delta(1232)$ mass region. These are very important quantities which arise in all viable quark, QCD, or perturbative QCD models of pion electroproduction and photoproduction.

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I. INTRODUCTION: THE $\Delta N\gamma$ TRANSITION FORM FACTORS AND MULTIPOLES

The $\Delta N\gamma$ transition form factors [1] $G_M^*(q^2)$, $G_E^*(q^2)$, and $G_C^*(q^2)$ and their multipole counterparts M_{1^+}, E_{1^+}, and S_{1^+} in nuclear and elementary particle physics are very important in nuclear and elementary particle physics because they provide a basis for testing theories of effective quark forces or production models.

- They are especially important in the understanding of perturbative QCD (PQCD) models [2] involving gluon exchange mechanisms, tensor interactions, or possible hybrid baryonic states;

- They are important in enhanced quark models in which the transition form factors may be calculated as a function of q^2; in electroproduction and photoproduction processes; in symmetry schemes such as SU(6) and U(6,6), and Melosh transformations; in bag models; in dispersion relation and Bethe-Salpeter approaches; in current algebra baryon sum rules; and in nonperturbative methods such as lattice QCD, QCD sum rules, and algebraic formulations.

- The fundamental reason that the transition form factors are such good QCD probes lies in the fact that in many quark, symmetry, and potential models, $G_E^*(q^2)$ and/or $G_C^*(q^2)$ are identically zero thus giving rise to pure magnetic dipole M_{1^+} transitions

- In the naïve quark model, it can be shown that the quantity $E_{1^+}/M_{1^+} = -G_E^*/G_M^* \equiv$ ratio of the electric quadrupole moment to the magnetic dipole moment $\equiv R_{EM} = 0$. Experimentally, however, the R_{EM} appears to be non-zero but small in magnitude and of the order of a few percent. Most analyses predict the R_{EM} to be small and negative at small momentum transfer, however a recent analysis [3] extracted the value $R_{EM}(q^2 = -3.2\ (GeV/c)^2) \approx (+6 \pm 2 \pm 3)\%$. Subsequently, another even more recent analysis [4]predicts that $R_{EM} = -(2.5 \pm 0.2 \pm 0.2)\%$ at the maximum of the $\Delta(1232)$ resonance and $R_{EM} = -3.5\%$ when background scattering amplitude contributions are taken into consideration.

- Clearly, the capability of any particular theoretical model (including ours) to predict accurately and precisely non-zero R_{EM} values of the right sign and magnitude for particular values of q^2 in agreement with experiment is critical. As has been noted, the R_{EM} ratio is especially effective for testing effective quark forces such as occur in QCD one-gluon exchange tensor interactions, various types of enhanced quark models, symmetry schemes such as $SU(6)$, $U(6,6)$, melosh transformations, dispersion relations and sum rules (where the Δ always plays an important role).

A. Importance of the $\mathbf{\Delta \rightarrow N + \gamma}$ Transition Form Factors

- Provide a basis for testing theories of effective quark forces and production models

- QCD: One gluon exchange mechanisms, tensor interactions, and possible hybrid baryonic states.

- Enhanced Quark Models: They should be capable of predicting accurately the Δ–N transition form factors as a function of q^2.
- Bag models of hadrons
- Current Algebra approaches to hadron physics
- Non-perturbative approaches to hadron physics such as lattice QCD
- Electroproduction and Photoproduction: Important for correct theoretical description.
- Symmetry Schemes: The Δ always plays an important role in models involving $SU(6)$, $U(6,6)$, etc., and melosh transformations.
- Dispersion relations: The Δ always plays an important role.
- Baryon Sum Rules: The Δ always plays an important role.

B. The $\mathbf{\Delta \to N + \gamma}$ Transition Form Factors and Transition Amplitudes

In general one may write for the $\Delta \to N+\gamma$ transition amplitude the following expression [1]:

$$\langle p(\vec{p},\lambda_p)|\, j_\mu(0)\, \left|\Delta^+(\vec{p}^*,\lambda_\Delta)\right\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{mm^*}{E_p E_\Delta}}\bar{u}_P(\vec{p},\lambda_P)\left[\Gamma_{\mu\beta}\right] u^\beta_\Delta(\vec{p}^*,\lambda_\Delta) \tag{1}$$

where

$$\begin{aligned}\Gamma_{\mu\beta} &= i\frac{3(m^*+m)}{2m}(G^*_M - 3G^*_E)\Theta^{-1}m^* q_\beta \epsilon_\mu(qp\gamma) \\ &- \frac{3(m^*+m)}{2m}(G^*_M + G^*_E)\Theta^{-1}[2\epsilon_{\beta\sigma}(p^*p)\epsilon^\sigma_\mu(p^*p)\gamma_5 - im^* q_\beta\epsilon_\mu(qp\gamma)] \\ &+ \frac{3(m^*+m)}{m}G^*_C\Theta^{-1}q_\beta[p\cdot q q_\mu - q^2 p_\mu]\gamma_5.\end{aligned} \tag{2}$$

In Eqs. (1) and (2), the electromagnetic current is denoted by j_μ, $q \equiv p^* - p$, p^* and p are the four-momenta of the Δ^+ and nucleon respectively. $\Theta^{-1} \equiv [((m^*+m)^2 - q^2)((m^*-m)^2 - q^2)]^{-1}$ is a kinematic factor which depends on q^2, m^* (the Δ^+ mass), and m (the proton mass); λ_P and λ_Δ are the helicities of the proton and Δ^+ respectively. We note that the first, second, and third terms in Eq.(2) induce transverse $\frac{1}{2}$ (h_3), transverse $\frac{3}{2}$ (h_2), and longitudinal helicity transitions (h_1) respectively in the rest frame of the Δ^+ isobar [1]. G^*_M, G^*_E, and G^*_C are related to the helicity form factors h_1, h_2, and h_3 by the relations:

$$\begin{aligned}h_3 &= -\frac{3(m^*+m)}{2m}(G^*_M - 3G^*_E) \\ h_2 &= -\frac{3(m^*+m)}{2m}(G^*_M + G^*_E) \\ h_1 &= \frac{3(m^*+m)}{m}G^*_C.\end{aligned} \tag{3}$$

For the transition amplitude governing the virtual process $p \to p + \gamma$, we have similarly

$$\langle p(\vec{s},\lambda)|\, j_\mu(0)\, \left|p(\vec{t},\lambda^*)\right\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{m^2}{E_{\vec{p}}E_{\vec{p}^*}}}\bar{u}_p(\vec{s},\lambda)\,[\Gamma_\mu]\, u_p(\vec{t},\lambda^*) \tag{4}$$

where

$$\Gamma_\mu = [1-\tilde{q}^2/(4m^2)]^{-1}[(i/(4m^2))G_M(\tilde{q}^2)\epsilon_\mu\left(\tilde{P}\tilde{q}\gamma\right)\gamma_5 + (1/(2m))G_E(\tilde{q}^2)\tilde{P}_\mu], \tag{5}$$

$\tilde{P}_\mu \equiv \widetilde{p}+\widetilde{p}^*$, $\widetilde{q} = \widetilde{p}^* - \widetilde{p}$ with $\widetilde{p}^* = (\widetilde{p}^{*0},\vec{t})$ and $\widetilde{p} = (\widetilde{p}^0,\vec{s})$. $G_M(\widetilde{q}^2)$ and $G_E(\widetilde{q}^2)$ in Eq. (5) are the familiar nucleon Sachs form factors.

C. Relationship Between the ΔNγ Form Factors and Multipoles

The magnetic, electric, and coulombic multipole transition moments given by $M_{1^+}(q^2)$, $E_{1^+}(q^2)$, and $S_{1^+}(q^2)$ can be written [5] in terms of $G^*_M(q^2)$, $G^*_E(q^2)$, and $G^*_C(q^2)$. Indeed one has

$$\begin{aligned} M_{1^+} &= \alpha_1\,\sqrt{Q^-}\,G^*_M \\ E_{1^+} &= \alpha_2\,\sqrt{Q^-}\,G^*_E \\ S_{1^+} &= \alpha_3\,Q^-\sqrt{Q^+}\,G^*_C \end{aligned} \tag{6}$$

where α_1, $\alpha_2 = -\alpha_1$, α_3 are functions of parameters governing the process $\Gamma(\Delta \to \pi N)$ (and in particular are dependent on the Δ mass m^*) and where $Q^\pm \equiv \sqrt{(m^* \pm m)^2 - q^2}$.

D. $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ Photon Decay Helicity Amplitudes

The total Δ radiative width $\equiv \Gamma^T_\gamma$, for decay into $p+\gamma$ is given by:

$$\Gamma^T_\gamma = \frac{mq_c^2}{2m^*\pi}\sum_{\lambda=\frac{1}{2},\frac{3}{2}} A^2_\lambda \tag{7}$$

where

$$A_{\frac{1}{2}} = -e\left(\frac{\sqrt{3}}{8}\right)\left[\frac{m^{*2}-m^2}{m^3}\right]^{\frac{1}{2}}\left[G^*_M(0) - 3G^*_E(0)\right], \tag{8}$$

$q_c^2 =$ CM momentum, and

$$A_{\frac{3}{2}} = -e\left(\frac{3}{8}\right)\left[\frac{m^{*2}-m^2}{m^3}\right]^{\frac{1}{2}}\left[G^*_M(0) + G^*_E(0)\right] \tag{9}$$

Experimentally [6],

$$A_{\frac{1}{2}} \cong (-140\pm 5)\times 10^{-3} GeV^{-1/2} \qquad \text{and} \qquad A_{\frac{3}{2}} \cong (-258\pm 6)\times 10^{-3} GeV^{-1/2} \tag{10}$$

II. CALCULATION

- Our treatment is non-perturbative and performed in a broken symmetry hadronic world [7]. We do not require the use of "mean" mass approximations;
- Physical masses are used at all times. Thus, G^*_E is not forced to equal zero as in the naïve quark model;
- Our treatment is completely relativistic. Current conservation is guaranteed. Additionally, the correct electromagnetic transition operator is used in all calculations;
- We use the infinite-momentum frame for calculations or equivalently— "infinite" Lorentz boosts that are not always in the z-direction, thus implicitly (and often explicitly) bringing into play Wigner rotations resulting in mixed helicity particle states.
- In order to proceed with the calculation of $G^*_M(q^2)$, and $G^*_E(q^2)$, we consider helicity states with $\lambda = \pm 1/2$ and $\lambda_\Delta = 1/2$ (i.e. spin flip and non-flip sum rules) and the non-strange ($S = 0$) $L = 0$ ground state baryons ($J^{PC} = \frac{1}{2}^+, \frac{3}{2}^+$). We will ultimately find two independent constraint equations which will then allow one to calculate $G^*_M(q^2)$, and $G^*_E(q^2)$, and their multipole counterparts.
- It is well-known [7] that if one defines the axial-vector matrix elements: $\langle p, 1/2| A_{\pi^+} |n, 1/2\rangle \equiv f = g_A(0)$, $\langle \Delta^{++}, 1/2| A_{\pi^+} |\Delta^+, 1/2\rangle \equiv -\sqrt{\frac{3}{2}}\, g$, and $\langle \Delta^{++}, 1/2| A_{\pi^+} |p, 1/2\rangle \equiv -\sqrt{6}\, h$, and applies asymptotic level realization to the chiral $SU(2) \otimes SU(2)$ charge algebra $[A_{\pi^+}, A_{\pi^-}] = 2V_3$, then $h^2 = (4/25) f^{\,2}$ (the sign of $h = +(2/5)f$, can be fixed by requiring that $G^*_M(0) > 0$)and $g = (-\sqrt{2}/5)f$.
- If one inserts the algebra $[j^\mu_3(0), A_{\pi^+}] = A^\mu_{\pi^+}(0)$ ($j^\mu \equiv j^\mu_3 + j^\mu_S$, where $j^\mu_3 \equiv$ isovector part of j^μ and j^μ_S is isoscalar) between the ground states $\langle B(\alpha, \lambda = \pm 1/2, \vec{s})|$and $\left|B'(\alpha, \lambda = 1/2, \vec{t}\,)\right\rangle$ with $|\vec{s}| \to \infty, \left|\vec{t}\,\right| \to \infty$, where $\langle B(\alpha)|$ and $|B'(\beta)\rangle$ are the following $SU_F(2)$ related combinations: $\langle p, n\rangle$, $\langle p, \Delta^0\rangle$, $\langle \Delta^{++}, p\rangle$, $\langle n, \Delta^-\rangle$, $\langle \Delta^{++}, \Delta^+\rangle$, $\langle \Delta^+, \Delta^0\rangle$, $\langle \Delta^0, \Delta^-\rangle$ and $\langle \Delta^+, n\rangle$, then one obtains (we use $< N| j^\mu_S| \Delta >= 0$) the following two independent sum rule constraints:

Spin Non-Flip Sum Rule

$$< p, 1/2, \vec{s}\,|j^\mu(0)|\, \Delta^+, 1/2, \vec{t} > \; = \frac{5\sqrt{2}}{4}\left\{ -\frac{\langle p, 1/2, \vec{s}|\, A^\mu_{\pi^+}(0)\, \left|n, 1/2, \vec{t}\right\rangle}{2f} + \left\langle p, 1/2, \vec{s}\,|j^\mu_3(0)|\, p, 1/2, \vec{t}\right\rangle \right\}. \tag{11}$$

and

Spin Flip Sum Rule

$$< p, -1/2, \vec{s}\,|j^\mu(0)|\, \Delta^+, 1/2, \vec{t} >= \frac{5}{8}\sqrt{2}\, \langle p, -1/2, \vec{s}|\, j^\mu_3(0)\, \left|p, 1/2, \vec{t}\right\rangle \tag{12}$$

Now take the limit $\left|\overrightarrow{t}\right| \to \infty$ and $|\overrightarrow{s}| =\to \infty$ and evaluate directly each of the matrix elements in Eq.(11) and Eq. (12). We find respectively that:

$$G_M^*(q^2) + \left[\frac{2m^*(4m - m^*) + m^2 - 2q^2}{2(m^{*2} + m^2 - q^2)}\right] G_E^*(q^2) = \tag{13}$$

$$\frac{5\sqrt{3}}{3}\left[\frac{2m^* m^2}{(m^* + m)(m^{*2} + m^2 - q^2)}\sqrt{\frac{(m^* + m)^2 - q^2}{4m^2 - q^2}}\right] G_M^V(q^2)$$

and

$$G_M^*(q^2) - 3G_E^*(q^2) = \left[\frac{5\sqrt{3}m\sqrt{-\tilde{q}^2}}{3(m^* + m)\left[(m^* - m)^2 - q^2\right]^{1/2}}\right] G_M^V(\tilde{q}^2), \tag{14}$$

where in Eq. (14), a collinear limit of $\left|\overrightarrow{t}\right|$ and $|\overrightarrow{s}|$ was taken in such a fashion that $|\overrightarrow{s}| = r\left|\overrightarrow{t}\right|$ ($\overrightarrow{s}$ and $\overrightarrow{t}$ are taken along the z-axis, $0 < r \leq m^2/m^{*2}$) and $\tilde{q}^2$ and q^2 are related by the equations $\tilde{q}^2 = (1 - r)\left[q^2 - (1 - r)m^{*2}\right]$ and $q^2 = \frac{(1-r)}{r}(m^{*^2} r - m^2)$.

Eq. (13) and Eq. (14) may be solved for $G_M^*(q^2)$ and $G_E^*(q^2)$. The numerical results (Figure 1.) are as follows:

III. RESULTS AND CONCLUSIONS

- $G_M^*(q^2)$, $G_E^*(q^2)$, $R_{EM}(q^2)$, $A_{\frac{1}{2}}$, and $A_{\frac{3}{2}}$ are computed in good agreement with experiment (Figure 1).

  1. At the Δ pole mass, we find that $G_M^*(0) = 3.18$, $G_E^*(0) = +0.07$, and $R_{EM}(0) = -2.19\%$. We also determine that $A_{\frac{1}{2}} \cong -134 \times 10^{-3} GeV^{-1/2}$, and $A_{\frac{3}{2}} \cong -254 \times 10^{-3} GeV^{-1/2}$.
  2. When the Δ mass is taken to be 1.232 GeV/c, $G_M^*(0) = 3.09$, $G_E^*(0) = +0.12$, and $R_{EM}(0) = -3.94\%$. We also determine that $A_{\frac{1}{2}} \cong -128 \times 10^{-3} GeV^{-1/2}$, and $A_{\frac{3}{2}} \cong -262 \times 10^{-3} GeV^{-1/2}$;

- We find that $R_{EM}(q^2)$ is negative for $0 \leq -q^2 \lesssim 5\ GeV^2/c^2$, changes sign in the region $-q^2 \approx 6 - 7\ GeV^2/c^2$, and very slowly approaches 1 as $-q^2 \to \infty$;

- The PQCD prediction that $R_{EM}(q^2) \to 1$ as $-q^2 \to \infty$ is verified but only as an asymptotic condition applicable only at very high momentum transfer [8];

- The R_{EM} is particularly sensitive to the Δ mass (i.e. mass parametrization used) when $0 \leq -q^2 \lesssim 1\ GeV^2/c^2$ (i.e. photoproduction).

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